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OSCILLATION FOR FIRST ORDER LINEAR DIFFERENTIAL EQUATIONS WITH DEVIATING ARGUMENTS*

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Abstract. Some new sufficient conditions for oscillation of first order linear delay differential equations, as well as neutral differential equations have been obtained. Forced oscillation for first order differential equations with deviating arguments has been studied also.

I. Introduction. In the past several years the problem of finding sufficient conditions for the oscillation of first order delay differential equations of the form

$$y'(t) + p(t)y(\tau(t)) = 0 \tag{1.1}$$

where p, τ are continuous on $[a, +\infty)$, $p(t) \geq 0$, $\tau(t) < t$ and $\lim_{t\to\infty} \tau(t) = \infty$ has received increasing attention [7,9]. It is known that every solution of (1.1) is oscillatory if either of the following conditions hold [1-5]:

(i) $$\liminf_{t\to\infty} \int_{\tau(t)}^{t} p(s)\,ds > \frac{1}{e} \tag{1.2}$$

(ii) $$\limsup_{t\to\infty} \int_{\tau(t)}^{t} p(s)\,ds > 1, \quad \tau'(t) \geq 0, \quad p(t) > 0. \tag{1.3}$$

Further generalizations of (1.2) have been given in [1]. In §II of this paper we shall give some results which improve (1.2) and (1.3). In §III we extend the results in §II to first order neutral differential equations and obtain some new oscillation criteria. Finally, in §IV we consider the forced equation of the form

$$y'(t) + p(t)y(\tau(t)) = F(t) \tag{1.4}$$

which has received much less attention. We obtain some simple criteria for oscillation of (1.4) which extend the results of Tomaras [8] and illustrate with some examples.

As usual, a solution of (1.1) (or (1.4)) is said to be oscillatory if it has arbitrarily large zeros and nonoscillatory if it is eventually positive or negative.

II. Oscillation of (1.1). We shall need the following lemma whose proof may be found in [3].

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