

Unitarity of the Knizhnik–Zamolodchikov–Bernard Connection and the Bethe Ansatz for the Elliptic Hitchin systems

Fernando Falceto¹, Krzysztof Gawędzki²

¹ Depto. Física Teórica, Univ. Zaragoza, E-50009 Zaragoza, Spain

² I.H.E.S., C.N.R.S., F-91440 Bures-sur-Yvette, France

Received: 24 April 1996 / Accepted: 7 June 1996

Abstract: We work out finite-dimensional integral formulae for the scalar product of genus one states of the group G Chern–Simons theory with insertions of Wilson lines. Assuming convergence of the integrals, we show that unitarity of the elliptic Knizhnik–Zamolodchikov–Bernard connection with respect to the scalar product of CS states is closely related to the Bethe Ansatz for the commuting Hamiltonians building up the connection and quantizing the quadratic Hamiltonians of the elliptic Hitchin system.

1. Introduction

The present paper continues the program [17, 10, 19, 20] aimed at analysis of the scalar product of states in the Chern–Simons (CS) theory. It extends the considerations of ref. [9] where we treated the $SU(2)$ CS theory on the elliptic curve (times the time-line and with insertions of time-like Wilson lines) to the case of a general group G . As in the previous papers of the series, the point is to express the formal scalar product of the CS theory, given by a functional integral over gauge fields, as a multiple finite-dimensional integral. The latter, if convergent for every state, provides the space of CS states \mathcal{W} with a Hilbert space structure and the holomorphic vector bundle \mathcal{W} , obtained by varying the modulus τ of the elliptic curve and the positions z_n of insertions, with a hermitian structure.

The integral expressions for the scalar product of the CS states are close cousins of the contour integral expressions for the conformal blocks of the corresponding WZW conformal theory. In the elliptic case, the contour integral representations were recently studied in ref. [12]. Our approach elucidates the origin of the complicated expressions which appear in such representations for a general group: they are induced by a simple trick, already used in [10], which handles a change of variables in the functional integral.

The WZW conformal blocks are holomorphic sections θ of the bundle \mathcal{W} of the CS state spaces satisfying in the elliptic case the Knizhnik–Zamolodchikov–Bernard