Half–sided Modular Inclusion
and the Construction of the Poincaré Group

H.J. Borchers

Institut für Theoretische Physik der Universität Göttingen, Bunsenstrasse 9,
D–37073 Göttingen, Germany. E-mail: Borchers@Theorie.Physik.UNI-Goettingen.DE

Abstract: In this note we will give a construction of the Poincaré group out of the modular groups of the wedge algebras provided the groups act on the algebra of every double cone like the associated Lorentz boosts. This construction will use the concept of half–sided modular inclusions instead of the first and second cohomology of the Poincaré group as used by Brunetti, Guido and Longo. By our method we obtain directly the Poincaré group and not its covering group.

1. Introduction

The modular groups of different subalgebras appearing in the theory of local observables have been used to construct the symmetry–group of the whole theory. There are two examples, namely the construction of the Möbius–group in chiral quantum field theory by H.–W. Wiesbrock [Wie2] and the construction of the Poincaré group in the four–dimensional theory of local observables due to Brunetti, Guido and Longo [BGL] and of Guido and Longo [GL].

   If one starts from two or more one–parametric groups, these groups will generally create an infinite dimensional group. In the above–mentioned examples, however, we obtain only a finite dimensional Lie–group and one is interested to understand the reason for this reduction. By superficial consideration one might think that the reduction is due to the locality of the action of the modular groups we are starting with. These locality assumptions are consequences of either the result of Bisognano and Wichmann [BW1,2] or of the conclusions of Borchers [Bch1]. But this alone does not lead to a reduction and I think that this reduction is due to the fact that there are pairs of algebras the intersection of which fulfills the condition of half–sided modular inclusion with respect to both algebras.

   With two algebras fulfilling the condition of half–sided modular inclusion one finds a group $U(t)$ which maps the large algebra onto the small one if t takes the value 1. This group has either a positive or a negative spectrum, from which one is usually able to show the uniqueness of this group. This uniqueness seems to be the