A Rohlin Property for One-Parameter Automorphism Groups

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Abstract: We define a Rohlin property for one-parameter automorphism groups of unital simple C^*-algebras and show that for such an automorphism group any co-cycle is almost a coboundary. We apply the same method to the single automorphism case and show that if an automorphism of a unital simple C^*-algebra with a certain condition has a central sequence of approximate eigen-unitaries for any complex number of modulus one, then any cocycle is almost a coboundary, or the automorphism has the stability. We also show that if a one-parameter automorphism group of a unital separable purely infinite simple C^*-algebra has the Rohlin property then the crossed product is simple and purely infinite.

1. Introduction

A C^*-dynamical system is a C^*-algebra with an action of a locally compact group by automorphisms. To analyse such a system the notion of Rohlin property was introduced and exploited at least when the group is the integer group $\mathbf{Z}$ or perhaps an amenable discrete group [8, 12, 13, 5, 3, 4, 22, 26, 19, 20]. We here introduce a Rohlin property for one-parameter automorphism groups; if α is a strongly continuous one-parameter automorphism group of a unital simple C^*-algebra A, α is said to have the Rohlin property if for any real number $p \in \mathbf{R}$ there is a central sequence $\{v_n\}$ of unitaries in A such that $\alpha_t(v_n) - e^{ipt}v_n$ converges to zero uniformly in t on every bounded subset of $\mathbf{R}$. In this case the spectral projections of v_n would be periodically transformed, in a sense, under α_t with period $2\pi/p$; so this is an analogue of the Rohlin property for single automorphisms. The main result (Theorem 2.1) will show that then for any α-cocycle u, i.e., a continuous family $u(t)$ of unitaries with $u(s)\alpha_s(u(t)) = u(s + t)$, $s, t \in \mathbf{R}$ is almost a coboundary, i.e., has a sequence $\{w_n\}$ of unitaries with $w_n\alpha_t(w_n^*) \to u(t)$ uniformly in t on every bounded subset of $\mathbf{R}$. (Here a small condition on u should be imposed; see 2.1 for details.) The only natural examples we can give of one-parameter automorphism groups with the Rohlin property are on simple non-commutative tori (Proposition 2.5). (Others may

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Dedicated to: Prof. H. Hasegawa