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Time Dependent Periodic Navier-Stokes Flows on a Two-Dimensional Torus

Zhi-Min Chen[*1], **W.G. Price**[2]

[1] Department of Mathematics, Tianjin University, Tianjin 300072, P.R. China. E-mail: zhimin@tju.edu.cn
[2] Department of Ship Science, Southampton University, Southampton SO17 1BJ, UK

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Abstract: The dynamical behaviour of an incompressible viscous fluid flow on a two-dimensional torus externally excited by a spatially periodic force is investigated. The flow field, described by Navier-Stokes equations, is found to possess a sequence of time-periodic solutions which bifurcate from a single steady state solution (i.e. Hopf bifurcations). This result is based on a combination of analysis and computations, and each provides corroborative evidence to the findings of the other.

1. Introduction

The Hopf bifurcation plays an important role in describing the behaviour of nonlinear phenomena occurring in the theory of dynamical systems (see for example, Marsden and McCracken (1976), Guckenheimer and Holmes (1983), Thompson and Stewart (1986)). In the flow of a viscous fluid, for example, it is commonly supposed to appear in transition from a steady state to a time dependent periodic flow. Landau (1944), Ruelle and Takens (1971), Newhouse et al. (1978) considered Hopf bifurcation to occur in the initial stage of transition from laminar to turbulent flows. However, from a strictly rigorous analytical viewpoint detailed studies of the existence of Hopf bifurcations in fluid motions are rather limited. The reason for this lies in the lack of efficient analysis methods to solve the problem, partly because the dynamical system modelling the fluid flow has an infinite number of degrees of freedom.

The Hopf bifurcation is central to the theme of this paper and through a combination of analysis with computation, we discuss its occurrence in an incompressible viscous fluid flow on a two-dimensional torus externally excited by a sinusoidal force. To do so, let $k \geq 1$ be a positive integer and $\mathscr{T}^2$ the two-dimensional torus $\mathscr{S}^1 \times \mathscr{S}^1$ with $\mathscr{S}^1$ the unit circle $\mathscr{R}/(2\pi\mathscr{Z})$. Thus, in an incompressible viscous fluid, we examine the behaviour of time dependent periodic Navier-Stokes flows excited by a spatially sinusoidal external body force $(4k^2 \sin 2ky, 0)$.

The dynamical behaviour of this fluid flow system defined in terms of velocity $u = (u_1, u_2)$ and pressure p is described by the Navier-Stokes equations

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