Commun Math Phys 179, 417–466 (1996)

Communications in Mathematical Physics

The Large Time Stability of Sound Waves

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Received: 4 August 1995 / Accepted: 21 November 1995

Abstract: We demonstrate the existence of solutions to the full 3×3 system of compressible Euler equations in one space dimension, up to an arbitrary time $T > 0$, in the case when the initial data has arbitrarily large total variation, and sufficiently small supnorm. The result applies to periodic solutions of the Euler equations, a nonlinear model for sound wave propagation in gas dynamics. Our analysis establishes a growth rate for the total variation that depends on a new length scale d that we identify in the problem. This length scale plays no role in 2×2 systems, (or any system possessing a full set of Riemann coordinates), nor in the small total variation problem for $n \times n$ systems, the cases originally addressed by Glimm in 1965. Recent work by a number of authors has demonstrated that when the total variation is sufficiently large, solutions of 3×3 systems of conservation laws can in general blow up in finite time, (independent of the supnorm), due to amplifying instabilities created by the non-trivial Lie algebra of the vector fields that define the elementary waves. For the large total variation problem, there is an interaction between large scale effects that amplify and small scale effects that are stable, and we show that the length scale on which this interaction occurs is d. In the limit $d \to \infty$, we recover Glimm's theorem, and we observe that there exist linearly degenerate systems within the class considered for which the growth rate we obtain is sharp.

1. Introduction

We consider the initial value problem for the system of compressible Euler equations in one space dimension,

* Supported in part by NSF Applied Mathematics grant numbers DMS-92-06631, DMS-95000694, in part by ONR, US Navy grant number N00014-94-1-0691, A Guggenheim fellowship, and by the Institute of Theoretical Dynamics, UC-Davis
** Partially supported by DOE grant number DE-FG02-88ER25053 while at the Courant Institute, and by NSF grant number DMS-9201581 and DOE grant number DE-FG02-90ER25084.