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Spreading of Wave Packets in the Anderson Model on the Bethe Lattice

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Abstract: The spreading of wave packets evolving under the Anderson Hamiltonian on the Bethe Lattice is studied for small disorder. The mean square distance travelled by a particle in a time t is shown to grow as t^2 for large t.

1. Introduction

The Anderson model [6] gives a description of the motion of a quantum-mechanical electron in a crystal with impurities. It is given by the random Schrödinger operator

$$H_\lambda = \tfrac{1}{2}\Delta + \lambda V \quad \text{on } l^2(\mathbb{L})\,; \tag{1.1}$$

where $\mathbb{L}$ is either $\mathbb{Z}^d$ or the Bethe lattice $\mathbb{B}$ (same as Cayley tree – an infinite connected graph with no closed loops and a fixed number $K+1$ of nearest neighbors at each vertex ($K \geqq 2$, so $\mathbb{B}$ is not the line $\mathbb{R}$); the distance between two sites x and y in $\mathbb{B}$ will be denoted by $d(x,y)$ and is equal to the length of the shortest path connecting x and y). The (centered) Laplacian Δ is defined by

$$(\Delta u)(x) = \sum_y u(y)\,, \tag{1.2}$$

where the sum runs over all nearest neighbors of x in $\mathbb{L}$, and V is a random potential, with $V(x)$, $x \in \mathbb{L}$, being independent, identically distributed random variables with common probability distribution μ. The real parameter λ is called the *disorder*.

It follows from ergodicity that the spectrum of the Hamiltonian H_λ is given by

$$\sigma(H_\lambda) = \sigma\left(\tfrac{1}{2}\Delta\right) + \lambda \operatorname{supp} \mu \tag{1.3}$$

with probability one [33,9,3], where $\sigma(\frac{1}{2}\Delta)$ equals $[-d,d]$ if $\mathbb{L} = \mathbb{Z}^d$ and $[-\sqrt{K}, \sqrt{K}]$ if $\mathbb{L} = \mathbb{B}$. The decomposition of $\sigma(H_\lambda)$ into pure point spectrum, absolutely

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