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$N = 2$ Structures on Solvable Lie Algebras: The $c = 9$ Classification

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Abstract: Let $\mathfrak{g}$ be a finite-dimensional Lie algebra (not necessarily semisimple). It is known that if $\mathfrak{g}$ is self-dual (that is, if it possesses an invariant metric) then it admits an $N = 1$ (affine) Sugawara construction. Under certain additional hypotheses, this $N = 1$ structure admits an $N = 2$ extension. If this is the case, $\mathfrak{g}$ is said to possess an $N = 2$ structure. It is also known that an $N = 2$ structure on a self-dual Lie algebra $\mathfrak{g}$ is equivalent to a vector space decomposition $\mathfrak{g} = \mathfrak{g}_+ \oplus \mathfrak{g}_-$, where $\mathfrak{g}_\pm$ are isotropic Lie subalgebras. In other words, $N = 2$ structures on $\mathfrak{g}$ are in one-to-one correspondence with Manin triples $(\mathfrak{g}, \mathfrak{g}_+, \mathfrak{g}_-)$. In this paper we exploit this correspondence to obtain a classification of the $c = 9$ $N = 2$ structures on solvable Lie algebras. In the process we also give some simple proofs for a variety of Lie algebraic results concerning self-dual Lie algebras admitting symplectic or Kähler structures.

1. Introduction

Finite-dimensional reductive – i.e., direct products of simple and abelian – Lie algebras lie at the heart of many constructions in conformal field theory, string theory, topological field theory, and two-dimensional quantum gravity. Many of the properties of these theories are governed in part by the existence of more complicated underlying algebraic structures which have come to be known loosely as chiral algebras, W-algebras, or vertex operator algebras (VOAs). It is widely believed that the classification of these algebraic structures would take us a good deal closer to a full understanding of the above mentioned physical theories; but alas this proves to be a difficult problem. Nevertheless, the available results suggest that reductive Lie algebras will play an important organizational role in this classification. Indeed one finds a variety of functorial constructions starting from (the loop algebra of) a reductive Lie algebra and resulting in one of these more complicated algebraic structures. The best known construction of this kind is perhaps the (affine) Sugawara construction [1]; but in fact, there are many more: the coset construction [2], the