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The Boltzmann–Grad Limit for a One-Dimensional Boltzmann Equation in a Stationary State

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Abstract: In this paper we consider a one-dimensional model of interacting particles in a bounded interval with (possibly not homogeneous) diffusive boundary conditions. We prove that, when the number of particles N goes to infinity and the interaction is suitably rescaled (the Boltzmann–Grad limit), the one-particle distribution function of the unique invariant measure for the particle system, converges to the unique solution of the Boltzmann equation of the model, provided that the mean free path is sufficiently large.

1. Introduction

One of the most important problems in nonequilibrium statistical mechanics is the analysis of stationary nonequilibrium states. For instance one can couple the system under consideration to thermal reservoirs which are maintained at different constant temperatures. After an initial transient time, the system is expected to approach a stationary state in which there is a steady flux of energy from the hot to the cold reservoir.

Such a state can be described at various levels: macroscopically by means of the hydrodynamical equations, mezoscopically, through the kinetic (Boltzmann) equations, microscopically according to the basic Liouville equation. The three levels of descriptions are related by various scaling limits some of which (see for instance [1] as regards the deduction of the stationary hydrodynamical solutions from the Boltzmann equation) have been successfully investigated. However very little is known about stationary nonequilibrium states in the microscopic description. Namely what we know about stationary solutions of the Liouville equation for realistic systems with a non-constant profile of temperature on the boundary is, at most, existence and uniqueness (see e.g. [2]), without any additional property which could allow to go further in analyzing the scaling limits. Even worse is our knowledge as regards the Boltzmann equation: we have only partial existence results (see [3–5]) for stationary nonequilibrium solutions. In this situation a rigorous analysis of the Boltzmann–Grad limit seems hopeless, that is a derivation of the stationary