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Fusion of the q-Vertex Operators and its Application to Solvable Vertex Models

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Abstract: We diagonalize the transfer matrix of the inhomogeneous vertex models of the 6-vertex type in the anti-ferroelectric regime directly in the infinite lattice. For this purpose we have introduced new types of q-vertex operators. The special cases of those transfer matrices were used to diagonalize the s-d exchange model [23, 1, 6]. New vertex operators are constructed from the level one vertex operators by the fusion procedure. Using this construction we determine the commutation relations among new vertex operators which play a crucial role for the diagonalization. In order to clarify the quasi-particle structure of the model we estabish new isomorphisms of crystals. The isomorphisms figure out, representation theoretically, the ground state degenerations.

1. Introduction

In [2] the anti-ferroelectric XXZ hamiltonian, or equivalently, the transfer matrix of the 6-vertex model has been diagonalized directly in the thermodynamic limit based on the quantum affine symmetry. The method is powerful enough, on the one hand, to give the integral formulas for correlation functions and form factors, on the other hand, to determine the physical space as a representation of a quantum affine algebra $U_q(\widehat{sl_2})$.

A similar approach is possible for several two dimensional lattice models such as the ABF model [11, 4]. Among them a direct generalization of the 6-vertex model is the vertex models associated with the perfect representations of any level [15, 16]. Although there are technical problems of bosonization in the case of higher levels, at least the strategy is clear and everything we need is in our hands.

In this paper I want to add one more class of vertex models which can be solved by a similar method and are not contained in the class of directly generalized models above. The models which we study here are the inhomogeneous vertex models of 6-vertex type with the inhomogeneities in the spins. Namely, on the infinite regular square lattice, with each horizontal and vertical line except a finite number of vertical lines $l_1, \ldots, l_n$, we associate the vector space $\mathbf{C}^2$. With