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On a Model for Quantum Friction, II. Fermi's Golden Rule and Dynamics at Positive Temperature

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Abstract: We investigate the dynamics of an N-level system linearly coupled to a field of mass-less bosons at positive temperature. Using complex deformation techniques, we develop time-dependent perturbation theory and study spectral properties of the total Hamiltonian. We also calculate the lifetime of resonances to second order in the coupling.

1. Introduction

Let $\mathscr{A}$ be a quantum mechanical N-level system with energy operator H_A on the Hilbert space $\mathscr{H}_A = \mathbf{C}^N$. We denote by $E_1 < E_2 < \cdots < E_M$ the eigenvalues of H_A listed in increasing order. We will colloquially refer to $\mathscr{A}$ as an *atom* or *small system*. Even though we formulate our results for the N-level system $\mathscr{A}$ most of them will, in some sense, extend to situations where $\mathscr{H}_A$ is infinite dimensional and H_A unbounded – see Remark 4 at the end of Sect. 2 for more details.

Let $\mathscr{B}$ be an infinite heat bath. In this paper $\mathscr{B}$ will be an infinite free Bose gas at inverse temperature $\beta = 1/kT$, without Bose–Einstein condensate. This system is described (see e.g. [BR, D1, D2, LP]) by a triple $\{\mathscr{H}_B, \Omega_B, H_B\}$, where $\mathscr{H}_B$ is a Hilbert space, H_B a self-adjoint operator on $\mathscr{H}_B$, and Ω_B a unit vector in $\mathscr{H}_B$. Let us denote by $\omega(\mathbf{k})$ the energy of a boson with momentum $\mathbf{k} \in \mathbf{R}^3$. Then the equilibrium momentum distribution of bosons at inverse temperature β is given by Planck's law,

$$\rho(\mathbf{k}) = \frac{1}{\exp(\beta\omega(\mathbf{k})) - 1} .$$

The space $\mathscr{H}_B$ carries a representation of Weyl's algebra (CCR),

$$W_B(f) = \exp(i\varphi_B(f)) , \tag{1.1}$$

where the field operators $\varphi_B(f)$ satisfy, for $(1 + \omega^{-1/2})f \in L^2(\mathbf{R}^3)$, the relation

$$(\Omega_B, W_B(f)\Omega_B) = \exp\left[-\frac{\|f\|^2}{4} - \frac{1}{2}\int_{\mathbf{R}^3} |f(\mathbf{k})|^2 \rho(\mathbf{k})\, d^3k\right] . \tag{1.2}$$