Commun. Math. Phys. 176, 577–617 (1996)

Communications in
Mathematical
Physics

Distribution of Eigenvalues for the Modular Group

E. Bogomolny★, F. Leyvraz★★, C. Schmit

Division de Physique Théorique, Unité de Recherche des Universités Paris 11 et Paris 6, Associée au CNRS, Institut de Physique Nucléaire, F-91406 Orsay Cedex, France

Received: 25 November 1994

Abstract: The two-point correlation functions of energy levels for free motion on the modular domain, both with periodic and Dirichlet boundary conditions, are explicitly computed using a generalization of the Hardy–Littlewood method. It is shown that in the limit of small separations they show an uncorrelated behaviour and agree with the Poisson distribution but they have prominent number-theoretical oscillations at larger scale. The results agree well with numerical simulations.

1. Introduction

Free motion on constant negative curvature surfaces (CNCS) generated by discrete groups is the oldest and in some sense the best example of classically chaotic motion (see e.g. [1–3]). In recent years this subject has attracted wide attention also within the context of quantum chaos. There the main question is the way in which classical chaos manifests itself in the properties of the corresponding quantum systems (see e.g. [4, 5]). An important property of CNCS models is the existence of an *exact* relationship between the density of eigenvalues of the Laplace–Beltrami operator on the surface (= energy levels) and the geodesic on the surface, which correspond to classical periodic orbits. This is known as the Selberg trace formula (see e.g. [6, 7]). For arbitrary systems, only an approximate connection of this type is known, namely the so-called Gutzwiller trace formula [8, 5], which is asymptotically valid in the limit of highly excited states but does not have a good estimate on the error. It is therefore the coexistence of hard classical chaos and the exact Selberg trace formula which makes the study of CNCS models so important.

The simplest hallmark of classical chaos for a quantum system is the nature of the spectral fluctuations of the energy levels. It was conjectured that, for ergodic

---

★On leave of absence from L.D. Landau Institute of Theoretical Physics, 142432 Cherogolovka, Russia

★★Permanent address: Instituto de Física, University of Mexico, Apdo. postal 20-364, 01000 Mexico City, Mexico