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A Fully Supersymmetric AKNS Theory

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Abstract: We construct a fully supersymmetric biHamiltonian theory in four superfields, admitting zero curvature and Lax formulation. This theory is an extension of the classical AKNS, which can be recovered as a reduction. Other supersymmetric theories are obtained as reductions of the susy AKNS, namely a nonlinear Schrödinger, a modified KdV and the Manin–Radul KdV. The susy nonlinear Schrödinger hierarchy is related to the one of Roelofs and Kersten; we determine its biHamiltonian and Lax formulation. Finally, we show that the susy KdV's mentioned before are related through a susy Miura map.

1. Introduction and Preliminaries

In the last decade there has been increasing interest in superextensions of the soliton evolution equations. The earlier results concerned the construction of field theories with fermionic and bosonic fields depending on time and one space variable x [Kup]. Next, the susy (=supersymmetric) soliton equations were investigated. In the so-called $N = 1$ susy extensions [MR], in which we are mainly interested, the field variables depend, apart from time, on the superspace variables x, θ, with x even and θ odd; the field equations are formulated in terms of the superderivative $D = \theta\partial/\partial x + \partial/\partial\theta$, with the property $D^2 = \partial/\partial x$. Also, the $N = 2$ susy extensions were introduced, with one even and two odd superspace variables x, θ_1, θ_2 [Mat]; in this case, two superderivatives $D_i = \theta_i\partial/\partial x + \partial/\partial\theta_i$ are employed.

The best known methods for constructing soliton equations can be appropriately generalized to the susy framework. The Lax formalism in terms of fractional powers was extended by introducing an algebra of pseudodifferential operators in D (or D_1 and D_2); in this way, some susy KdV equations were constructed [MR, Mat, LM], and the corresponding biHamiltonian structures were obtained via R-matrix theory [OP, FMR].

Moreover, the connections between the susy soliton equations and the theory of Lie superalgebras were analyzed in [IK1–4, MP1], in order to obtain a superanalogue of the classical Drinfeld–Sokolov theory [DS].