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Dimensions of Invariant Sets of Expanding Maps

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Abstract: We consider a compact invariant set Λ of an expanding map of a manifold M and give upper and lower bounds for the Hausdorff Dimension $\dim_H(\Lambda)$, and box dimensions $\underline{\dim}_B(\Lambda)$ and $\overline{\dim}_B(\Lambda)$. These bounds are given in terms of the topological entropy, topological pressure, and uniform Lyapunov exponents of the map.

A measure-theoretic version of these results is also included.

0. Introduction

It is well known that many self-similar sets, or fractals, can be realized as invariant sets of smooth expanding maps. The purpose of this paper is to give estimates on the box dimensions and Hausdorff dimensions of these sets.

Most of the known results on dimensions in dynamical systems are of the following types. Some are about the dimensions of invariant measures (see e.g. [L] and [LY]). Some are about the dimensions of sets constructed via a sequence of affine contractions (e.g. [F1] and [GL]). Others deal with subsets of the line. See also [G, MM, BU], etc.

At present, there are few known effective ways to calculate the box and Hausdorff dimensions of *sets* (sets, not measures) invariant under *nonlinear* maps in dimensions larger than 1. In this paper we consider invariant sets of uniformly expanding maps. We give upper and lower bounds for box and Hausdorff dimensions in terms of topological entropy, topological pressure, and uniform Lyapunov exponents. Examples of self-affine sets show that these bounds are sharp (see Sect. 1, Example 2). We will also give parallel results for dimensions of invariant measures.

We state our results.

Let M be an m-dimensional C^∞ Riemannian manifold with Riemannian measure ν, and f be a C^2 map from M to itself. Let $\Lambda \subset M$ be an f-invariant compact subset, where f-invariance means $f\Lambda = \Lambda$. We also assume that f is expanding

★ Part of this work was done when I was in the Department of Mathematics, University of Arizona.