

Critical Circle Maps Near Bifurcation

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Abstract: We estimate harmonic scalings in the parameter space of a one-parameter family of critical circle maps. These estimates lead to the conclusion that the Hausdorff dimension of the complement of the frequency-locking set is less than 1 but not less than $1/3$. Moreover, the rotation number is a Hölder continuous function of the parameter.

1. Preliminaries

1.1. Introduction. This paper will present results about circle maps and families of circle maps that we were able to obtain during the past couple of years. We will not discuss diffeomorphisms, which by far and large are the best understood class of circle maps. In the present paper, we will deal with critical homeomorphisms. Some methods and estimates can be carried over to non-invertible maps, but we only mention [4] here.

Let us start by defining the class of maps we consider.

The objects that we intend to investigate. Points of the real line can be projected onto the unit circle in the complex plane by means of the map

$$x \rightarrow \exp(2\pi i x).$$

Maps from the real line project on the circle if they satisfy

$$f(x+1) - f(x) \in \mathbf{Z}$$

for every real x . Obviously, for a continuous map this difference must be constant, and is the topological degree of the circle map.

Unless necessary, we will not make a strong distinction between objects that live on the circle and their lifts to the universal cover. Whenever we want to make

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