Commun. Math. Phys. 174, 561–604 (1996)

Communications in Mathematical Physics

Combinatorial Quantization of the Hamiltonian Chern–Simons Theory II

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Received: 9 September 1994/in revised form: 31 January 1995

Abstract: This paper further develops the combinatorial approach to quantization of the Hamiltonian Chern Simons theory advertised in [1]. Using the theory of quantum Wilson lines, we show how the Verlinde algebra appears within the context of quantum group gauge theory. This allows to discuss flatness of quantum connections so that we can give a mathematically rigorous definition of the algebra of observables $\mathscr{A}_{CS}$ of the Chern Simons model. It is a $*$-algebra of "functions on the quantum moduli space of flat connections" and comes equipped with a positive functional ω ("integration"). We prove that this data does not depend on the particular choices which have been made in the construction. Following ideas of Fock and Rosly [2], the algebra $\mathscr{A}_{CS}$ provides a deformation quantization of the algebra of functions on the moduli space along the natural Poisson bracket induced by the Chern Simons action. We evaluate a volume of the quantized moduli space and prove that it coincides with the Verlinde number. This answer is also interpreted as a partition partition function of the lattice Yang–Mills theory corresponding to a quantum gauge group.

1. Introduction

This paper is a second part of the series devoted to combinatorial quantization of the Hamiltonian Chern Simons theory. Here we continue and essentially complete the analysis started in [1].

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Supported by Swedish Natural Science Research Council (NFR) under the contract F-FU 06821-304 and by the Federal Ministry of Science and Research, Austria.
** Part of project P8916-PHY of the 'Fonds zur Förderung der wissenschaftlichen Forschung in Österreich'; e-mail: grosse@pap.univie.ac.at.
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