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On the Molecular Limit of Coulomb Gases

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Abstract: We give a partially new analysis of the molecular nature of matter. A key feature is a property of the Coulomb potential as $\mathbb{R}^3$ is decomposed into simplices. A further application thereof is given in an appendix.

1. Introduction

A mixture of electrons and various kinds of nuclei consists of individual atoms and molecules, provided the temperature and the density are sufficiently low. Put differently, a gas of elementary particles is effectively described in this thermodynamic regime in terms of an ideal gas of composite particles. Different mathematical formulations and verifications of this fact have been given by Fefferman [4], by Conlon, Lieb and Yau [2], and by Macris and Martin [8]. See also [9, 10] for a discussion of the issues involved. With the present work we merely intend to offer a partially new proof. The reader familiar with the subject should proceed directly to Sect. 2.

The mixture shall consist of S species of spinless particles with masses $\mathbf{M} = (M_1, \dots, M_S)$ and charges $\mathbf{Q} = (Q_1, \dots, Q_S) \in \mathbb{Z}^S$. We assume that all negatively charged particles are fermions, whereas the statistics of the other particles is irrelevant. Let $N_k \in \mathbb{N}$ be the number of particles of the k^{th} species, and set $\mathbf{N} = (N_1, \dots, N_S)$. The total number of particles is $N = \sum_{k=1}^S N_k$. The Hilbert space $\mathscr{H}_{\mathbf{N},\Lambda}$ for $\mathbf{N}$ particles confined to an open set $\Lambda \subset \mathbb{R}^3$ is the subspace of $L^2(\Lambda)^{\otimes N}$ carrying the permutation symmetry appropriate to the given statistics. The Hamiltonian is

$$H_{\mathbf{N},\Lambda} = -\sum_{i=1}^{N} \frac{\Delta_{\Lambda,i}}{2m_i} + \sum_{\substack{i,j=1 \\ i<j}}^{N} \frac{q_i q_j}{|x_i - x_j|} =: T_{\mathbf{N},\Lambda} + V_{\mathbf{N}}\,, \tag{1.1}$$

where $(m_i, q_i) = (M_k, Q_k)$ if the i^{th} particle belongs to the k^{th} species. Here Δ_Λ is the Dirichlet Laplacian on Λ. If $\Lambda = \mathbb{R}^3$, the index Λ is omitted. Variable particle numbers are accounted for by means of the Fock space and the Hamiltonian

$$\mathscr{H}_\Lambda = \mathscr{F}(L^2(\Lambda)) = \bigoplus_{\mathbf{N}} \mathscr{H}_{\mathbf{N},\Lambda}\,, \qquad H_\Lambda = \bigoplus_{\mathbf{N}} H_{\mathbf{N},\Lambda}\,. \tag{1.2}$$