

# On the Quotients of Cubic Hecke Algebras

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**Abstract:** Between the rank 3 quotients of cubic Hecke algebras there is essentially one of maximal dimension. We prove it has a unique Markov trace having values in a torsion module. Therefore the description of a Markov trace on the cubic Hecke algebra corresponding to  $x^3 + 1$  and having the parameters  $(1, 1)$  is derived. Thus we obtain a numerical link invariant of finite degree, and define a whole sequence of 3<sup>rd</sup> order Vassiliev invariants.

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## 1. Introduction

The aim of this paper is to begin a systematic study of cubic Hecke algebras by analogy with the analysis carried out by Vaughan Jones (see [Jon87]) in the classical case of Hecke algebras. The motivation is to derive link invariants and Markov traces on the group algebra of the braid group.

We recall that Artin's braid group  $B_n$  in  $n$  strings is presented usually as

$$B_n = \langle b_1, b_2, \dots, b_{n-1} \mid b_i b_j = b_j b_i, \quad |i - j| > 1, \quad i, j = 1, n - 1; \\ b_{i+1} b_i b_{i+1} = b_i b_{i+1} b_i, \quad i = 1, n - 2 \rangle.$$

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