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Inverse Scattering Problem for the Schrödinger Equation with Magnetic Potential at a Fixed Energy

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Abstract: In this article we consider the Schrödinger operator in $R^n, n \geqq 3$, with electric and magnetic potentials which decay exponentially as $|x| \to \infty$. We show that the scattering amplitude at fixed positive energy determines the electric potential and the magnetic field.

1. Introduction

Consider the Schrödinger equation in R^n, $n \geqq 3$, with magnetic potential $A(x) = (A_1(x), \ldots, A_n(x))$ and electric potential $V(x)$:

$$-\sum_{j=1}^{n} \left(\frac{\partial}{\partial x_j} + iA_j(x) \right)^2 u + V(x)u = k^2 u \,, \tag{1}$$

$k > 0$, or equivalently

$$-\Delta u - 2i \sum_{j=1}^{n} A_j(x) \frac{\partial u}{\partial x_j} + q(x)u = k^2 u \,, \tag{1'}$$

where

$$q(x) = \sum_{j=1}^{n} \left(A_j^2(x) - i \frac{\partial A_j}{\partial x_j} \right) + V(x) \,. \tag{2}$$

We will assume that the potentials A and V are real-valued and exponentially decreasing, i.e.

$$\left| \frac{\partial^\alpha V(x)}{\partial x^\alpha} \right| \leqq C_\alpha e^{-\delta|x|}, \qquad \left| \frac{\partial^\beta A_j}{\partial x^\beta} \right| \leqq C_\beta e^{-\delta|x|}, \quad j = 1, \ldots, n \,, \tag{3}$$

for $0 \leqq |\alpha| \leqq P, 0 \leqq |\beta| \leqq P + 1$, where $P = n + 4$. We consider the solutions of (1) of the form

$$u = e^{ik\omega \cdot x} + v(x, \omega, k) \,, \tag{4}$$

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