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Bispectral KP Solutions and Linearization of Calogero–Moser Particle Systems

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Abstract: Rational and soliton solutions of the KP hierarchy in the subgrassmannian Gr_1 are studied within the context of finite dimensional dual grassmannians. In the rational case, properties of the tau function, τ, which are equivalent to bispectrality of the associated wave function, ψ, are identified. In particular, it is shown that there exists a bound on the degree of all time variables in τ if and only if ψ is a rank one bispectral wave function. The action of the bispectral involution, β, in the generic rational case is determined explicitly in terms of dual grassmannian parameters. Using the correspondence between rational solutions and particle systems, it is demonstrated that β is a linearizing map of the Calogero-Moser particle system and is essentially the map σ introduced by Airault, McKean and Moser in 1977 [2].

1. Introduction

Among the surprises in the history of rational solutions of the KP hierarchy (and the PDE's which make it up) are the existence of rational initial conditions to a non-linear evolution equation which remain rational for all time [1, 2], that these solutions are related to completely integrable systems of particles [2, 6, 7], and that a large class of wave functions which have been found to have the bispectral property turn out to be associated with potentials that are rational KP solutions [3, 16, 17]. Within the grassmannian which is used to study the KP hierarchy, the rational solutions, along with the N-soliton solutions, reside in the subgrassmannian Gr_1 [13]. This paper develops a general framework of finite dimensional grassmannians for studying the KP solutions in Gr_1 and then applies this to the bispectral rational solutions. New results include information about the geometry of KP orbits in Gr_1 and identification of properties equivalent to bispectrality. In addition, an explicit description of the bispectral involution in terms of dual grassmannian coordinates leads to the conclusion that it is, in fact, essentially the linearizing map σ [2].

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