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Character and Determinant Formulae of Quasifinite Representation of the $W_{1+\infty}$ Algebra

H. Awata[1], M. Fukuma[2], Y. Matsuo[3], S. Odake[4]

[1] Research Institute for Mathematical Sciences, Kyoto University, Kyoto 606-01, Japan. E-mail: awata@kurims-kyoto-u.ac.jp

[2] Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606, Japan. E-mail: fukuma@yukawa.kyoto-u.ac.jp

[3] Uji Research Center, Yukawa Institute for Theoretical Physics, Kyoto University, Uji 611, Japan. E-mail: yutaka@yukawa.kyoto-u.ac.jp

[4] Department of Physics, Faculty of Liberal Arts, Shinshu University, Matsumoto 390, Japan. E-mail: odake@jpnyitp.yukawa.kyoto-u.ac.jp

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Abstract: We diagonalize the Hilbert space of some subclass of the quasifinite module of the $W_{1+\infty}$ algebra. States are classified according to their eigenvalues for infinitely many commuting charges and the Young diagrams. The parameter dependence of their norms is explicitly derived. The full character formulae of the degenerate representations are given as summation of the bilinear combinations of the Schur polynomials.

1. Introduction

The detailed study of (infinite dimensional) Lie algebras has been sometimes very essential in theoretical physics. The representation theory of finite dimensional Lie algebra is indispensable to understand quantum mechanics or gauge theories. If we extend the dimension by one, the loop algebras such as Virasoro [1] or Kac–Moody algebras are essential tools to describe two-dimensional statistical systems or string theories.

Recently, in many places such as two-dimensional quantum gravity [2–5], the quantum Hall effects [6, 7], the membrane [8, 9], or the large N QCD [10, 11], the $W_{1+\infty}$ algebra is regarded as the fundamental symmetry of system.

As a member of loop algebras, the $W_{1+\infty}$ algebra has a unique character in that the number of currents is infinite. In a sense, it may be regarded as the symmetry of three-dimensional system since it is closely connected with the area-preserving diffeomorphism [12, 13]. Due to this fact, the detailed representation theory was not fully developed until now although some attempts were made [14]. The situation is also similar in the extensions of the $W_{1+\infty}$ algebra [15–20]. One of the confusing features of the $W_{1+\infty}$ algebra is its hybrid nature in dimensions. We remark that it has also definite "two-dimensional" aspects since we already knew the explicit realization in terms of two-dimensional free fields [13, 21]. Furthermore, this symmetry is found even in instanton physics in four dimensions [22–24].

Last year, Kac and Radul [25] discovered a way to avert from the difficulty and proved that the Hilbert space at each energy level can be finite dimensional if we choose the weight vector properly. In our previous letter [26], we give the computer