Commun. Math. Phys. 172, 287 – 292 (1995)

Communications in
Mathematical
Physics


# Leaf-Preserving Quantizations of Poisson $SU(2)$ are not Coalgebra Homomorphisms

**Albert J.L. Sheu***

Department of Mathematics, University of Kansas, Lawrence, KS 66045, USA.
email: sheu@kuhub.cc.ukans.edu

Received: 30 December 1993/in revised form: 1 December 1994

**Abstract:** Although it has been found that some deformation quantizations of the Poisson $SU(2)$ preserve symplectic leaves and some preserve the group (i.e. coalgebra) operation, this paper shows that a quantization of $SU(2)$ cannot be both leaf-preserving and group-preserving.

## 1. Introduction

Among various examples in the theory of quantum groups, one of the most well known and well understood in both algebraic and analytic contexts is quantum $SU(2)$, which has been studied in many different aspects [5, 15, 17].

We recall that on $SU(2)$ there is a multiplicative Poisson structure and Drinfeld's work shows that quantum $SU(2)$ gives a consistent algebraic deformation quantization of both the Poisson structure and the group structure. On the other hand, there have been found two types of C*-algebraic deformation quantization [13, 14, 1], a concept introduced by Rieffel [10, 11, 12], of the multiplicative Poisson structure on $SU(2)$ which are compatible with Woronowicz's C*-algebraic deformation quantization of the group structure of $SU(2)$ [17], in the sense that the C*-algebras obtained in these two processes are isomorphic. These two types of C*-algebraic deformations of the Poisson structure have very different features. One is constructed in a geometrically natural way (inspired by the concept of foliation C*-algebras [4]) and is a leaf-preserving deformation (to be defined later), while the other based on the work [6] is constructed in a more algebraic way and is not a leaf-preserving deformation, but it deforms the generators in Woronowicz's way and is actually a coalgebra isomorphism.

Since the coalgebra structure gives the group (action) structure, a deformation like the latter one which respects the coalgebra structure is probably of more interest from the algebraic viewpoint. It is a very interesting question to see whether the former quantization is actually a coalgebra isomorphism, and if not, whether there exists a leaf-preserving quantization that also preserves the coalgebra structure [12].

* Partially supported by NSF-Grant DMS-9002895 and DMS-9303231