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Non-Compact Quantum Groups Associated with Abelian Subgroups

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Abstract: Let G be a Lie group. For any Abelian subalgebra $\mathfrak{h}$ of the Lie algebra $\mathfrak{g}$ of G, and any $r \in \mathfrak{h} \wedge \mathfrak{h}$, the difference of the left and right translates of r gives a compatible Poisson bracket on G. We show how to construct the corresponding quantum group, in the C^*-algebra setting. The main tool used is the general deformation quantization construction developed earlier by the author for actions of vector groups on C^*-algebras.

Introduction

Let G be a connected Lie group with Lie algebra $\mathfrak{g}$, let H be a closed connected Abelian subgroup of G with Lie algebra $\mathfrak{h}$, and let $r \in \mathfrak{h} \wedge \mathfrak{h}$. Given this data, there is a simple way to equip G with a compatible Poisson bracket in Drinfeld's sense [D1, D3]. But according to Drinfeld's outlook, compatible Poisson brackets give precisely the "directions" in which one can hope to quantize a Lie group to obtain a quantum group. We will show here that, for the data given above, there is a natural way to carry out this quantization so as to construct the corresponding quantum group, within the framework of C^*-algebras. This quantization will be a *preferred* quantization in the sense of [GS1, GS2, Gq], meaning that the comultiplication will be unchanged – only the pointwise product of functions on G will be changed, into a non-commutative product. In the monograph [Rf2] I developed a general construction for the deformation quantization of any C^*-algebra in the direction of any Poisson bracket coming from an action of $\mathbb{R}^d$ for $d \geqq 2$. It is this construction which we will employ here to produce our quantum groups.

In [Rf4] I have described the details of this construction for the case in which G is compact, where there are some very substantial simplifications. This provided some new examples of compact quantum groups. Here we will see that our construction yields some reasonably interesting non-compact quantum groups which do not seem to have been constructed before. Since non-compact quantum groups have been

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