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On Operad Structures of Moduli Spaces and String Theory

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To the memory of Ansgar Schnizer

Abstract: We construct a real compactification of the moduli space of punctured rational algebraic curves and show how its geometry yields operads governing homotopy Lie algebras, gravity algebras and Batalin–Vilkovisky algebras. These algebras appeared recently in the context of string theory, and we give a simple deduction of these algebraic structures from the formal axioms of conformal field theory and string theory.

This paper started as an attempt to organize geometrically various algebraic structures discovered in 2d quantum field theory, see Witten and Zwiebach [46], Zwiebach [49], Lian and Zuckerman [29], Getzler [14, 15], Penkava and A. S. Schwarz [32], Horava [20], Getzler and J. D. S. Jones [16], Stasheff [43, 44] and Huang [21]. A more detailed version is available as hep-th 9307114.

The physical importance of these structures is that they lead toward the classification of string theories at the tree level, because the structure constants of the algebras appear as all correlators of the theory. We suggest that an appropriate background for putting together those algebraic structures is the structure of an operad. On the one hand, as we point out, a conformal field theory at the tree level is equivalent to an algebra over the operad of Riemann spheres with punctures, cf. Huang and Lepowsky [21, 22]. On the other hand, this one operad gives rise to several other operads creating these various algebraic strucutres. The relevance to physical is that theories such as conformal field theory or string-field theory provide a representation of the geometry of the moduli space of such punctured Riemann spheres in the category of differential graded vector spaces.

This paper, one of a series, deals with a part of these algebraic structures, namely with the structure of a homotopy Lie algebra and the related structures of the gravity algebra and Batalin–Vilkovisky algebra. A richer structure, the moduli space of Riemann spheres, induces a homotopy version of a Gerstenhaber algebra,

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