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Perturbative Quantum Field Theory at Positive Temperatures: An Axiomatic Approach

O. Steinmann
Universität Bielefeld, Fakultät für Physik, D-33501 Bielefeld, Germany

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Abstract: It is shown that the perturbative expansions of the correlation functions of a relativistic quantum field theory at finite temperature are uniquely determined by the equations of motion and standard axiomatic requirements, including the KMS condition. An explicit expression as a sum over generalized Feynman graphs is derived. The canonical formalism is not used, and the derivation proceeds from the beginning in the thermodynamic limit. No doubling of fields is invoked. An unsolved problem concerning existence of these perturbative expressions is pointed out.

1. Introduction

The traditional way of describing thermal equilibrium states of an infinitely extended quantum system, in particular of a quantum field theory, begins by restricting the system to a finite volume V, defining the canonical or grand canonical equilibrium by means of the familiar density matrices, and then going to the limit $V \to \infty$ (the "thermodynamic limit") for the quantities for which this limit can be expected to exist [1, 2]. This applies especially to the correlation functions of the fields and closely related objects like the expectation values of time ordered field products. Up to now most actual calculations of such functions have been based on this approach, using a Hamiltonian or Lagrangian formalism at finite V.

Another description of equilibria and their local disturbances, which can be used directly in the thermodynamic limit, has been developed in the framework of the algebras of local observables [3]. In this approach equilibrium states are characterized through an analyticity requirement for correlation functions, the so-called KMS condition. In the present paper we intend to show that this axiomatic method, suitably adapted to a field theoretical context, is perfectly capable of handling dynamical problems. More exactly, it will be shown that perturbative expansions for the correlation functions of a relativistic field theory, and related functions, can be derived directly in the thermodynamic limit, not making use of the canonical formalism, but using as only inputs the equations of motion and the axiomatic requirements that the correlation functions must satisfy. The result is represented as a sum over