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Deformation Quantization of the Heisenberg Group

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Abstract. A $*$-product compatible with the comultiplication of the Hopf algebra of the functions on the Heisenberg group is determined by deforming a coboundary Lie-Poisson structure defined by a classical r-matrix satisfying the modified Yang-Baxter equation. The corresponding quantum group is studied and its R-matrix is explicitly calculated.

1. Introduction

The quantization of a dynamical system on a symplectic manifold was introduced in [1, 2] by deforming the pointwise multiplication of the commutative algebra of the classical observables into a one parameter family $*_\hbar$ of associative but not necessarily commutative products. The parameter $\hbar$ is physically interpreted as the Planck constant and the deformation is required to satisfy the classical limit conditions

$$\phi *_\hbar \psi \underset{\hbar\to 0}{\longrightarrow} \phi\psi\,, \qquad (\phi *_\hbar \psi - \psi *_\hbar \phi)/\hbar \underset{\hbar\to 0}{\longrightarrow} \{\phi, \psi\}$$

for any pair of observables ϕ, ψ.

Since its first appearance, the method has found a constantly increasing number of applications and special attention has been devoted to systems with symmetry. For instance the geometric quantization, or coadjoint orbit method, yielding the irreducible representations of nilpotent Lie groups has been reproduced in this approach [3]; the deformation of quotient manifolds of the Heisenberg group by appropriate lattice subgroups has been investigated in [4] in connection with results on quantum tori; a framework for quantizing the linear Poisson structures has been proposed in [5]. Almost all deformations are expressed in terms of formal power series in $\hbar$ with coefficients in the algebra of the observables and a very tiny number of $*_\hbar$-products is explicitly known, the most relevant of which is obtained by the Weyl quantization on $\mathbf{R}^{2n}$. In [4, 5] the convergence of the power series is discussed and an answer is provided in terms of Fourier transforms.