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Local Instability of Orbits in Polygonal and Polyhedral Billiards

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Abstract. We classify when local instability of orbits of closeby points can occur for billiards in two dimensional polygons, for billiards inside three dimensional polyhedra and for geodesic flows on surfaces of three dimensional polyhedra. We sharpen a theorem of Boldrighini, Keane and Marchetti. We show that polygonal and polyhedral billiards have zero topological entropy. We also prove that billiards in polygons are positive expansive when restricted to the set of non-periodic points. The methods used are elementary geometry and symbolic dynamics.

1. Introduction

We consider billiards inside polygons and polyhedra. Such billiards have been well studied; good surveys are given in the references [Gu2, GaZ and CGa]. None-the-less, many fundamental questions about these systems remain unanswered: are billiards flows in polygons (polyhedral) ergodic, do they all have periodic points, etc.? In this article we study the topological dynamics of polygonal (polyhedral) billiards. The main tool used is coding of orbits by the sequence of edges (faces) they hit. A similar coding was considered in [K]. In the two dimensional case we prove that if two points code to the same forward sequence then they are both periodic. As a corollary we get a strengthening of a result of Katok [K], polygonal billiards have zero topological entropy. As a second corollary we get that any point is either periodic or the closure of its forward orbit includes a vertex, which gives a full topological classification of an a.e. result in [BKM]. Furthermore we show that the billiard flow, when restricted to the set of non-periodic points is positively expansive. These results hold for geodesic flows on polyhedra as well. In the convex three dimensional case more complicated behavior can arise. Every periodic symbolic sequence corresponds to at least one periodic billiard trajectory, however they can correspond to some quasi-periodic billiard trajectories as well. Nonperiodic symbolic sequences do not necessarily determine a unique billiard trajectory. None-the-less we get as a corollary to our results that polyhedral billiards have zero topological entropy and that any