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The Galilean Group in 2 + 1 Space-Times and its Central Extension

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Abstract: The problem of constructing the central extensions, by the circle group, of the group of Galilean transformations in two spatial dimensions; as well as that of its universal covering group, is solved. Also solved is the problem of the central extension of the corresponding Lie algebra. We find that the Lie algebra has a three parameter family of central extensions, as does the simply-connected group corresponding to the Lie algebra. The Galilean group itself has a two parameter family of central extensions. A corollary of our result is the impossibility of the appearance of non-integer-valued angular momentum for systems possessing Galilean invariance.

Ever since the pioneering work of Wigner [1] it has been appreciated that the representations of a symmetry group that are appropriate to quantum physics are the projective unitary (or anti-unitary) representations. That is, representations in a projective space P of a separable Hilbert space H that describes the state-space of a quantum-mechanical system. This idea also finds a reflection in the domain of classical mechanics [2]. Indeed, Wigner showed us how to understand the appearance of spin one-half particles in terms of the projective unitary irreducible representations of the Poincaré group. It also meanwhile became clear that the projective representations of a group are constructible from a knowledge of the ordinary (linear) representations of an associated group, which is the central extension of the original group by the circle group. Thus Bargmann [3] carried out his path-breaking analysis of the projective representations of continuous groups; in particular, of the Galilean group in $(3 + 1)$ space-times and showed how the concept of mass, with its associated superselection rule, arises via the central extension of the Galilean group. Later authors [4] provided further elaboration of the projective representations of the Galilean group as well as of the concept of a non-relativistic zero-mass system [4, 5].

The aim of the present paper is to solve the problem of finding central extensions of the proper Galilean group in $(2 + 1)$ space-time dimensions. There are several reasons for studying this problem. First, it is intrinsically interesting; the structure of the $(2 + 1)$ dimensional Galilean group is significantly different from that of the