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Vortex Condensation in the Chern–Simons Higgs Model: An Existence Theorem

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Abstract: It is shown that there is a critical value of the Chern–Simons coupling parameter so that, below the value, there exists self-dual doubly periodic vortex solutions, and, above the value, the vortices are absent. Solutions of such a nature indicate the existence of dyon condensates carrying quantized electric and magnetic charges.

1. Introduction

In this paper, we shall concentrate on a mathematically simplified anyon model known as the Abelian Chern–Simons Higgs model in which the Yang–Mills or the Maxwell field term is dropped and the Higgs potential takes a specific form. The dominance of the Chern–Simons gauge field gives rise to both electrically and magnetically charged vortices known as dyons. In the recent work [HKP, JW], it is found that there exists a self-dual structure so that the model permits a class of topological multivortex solutions with quantized charges similar to the solutions in the Abelian Higgs equations [JT, T1, T2] and a class of nontopological solutions carrying fractional values of charges [CHMcY, JLW, SY1]. This raises hope to establish the existence of condensates or periodic multivortices in the model. In this paper, we will present such a result. Note that the vortex condensation phenomenon in a gauged nonlinear Schrödinger equation has been established in [O]. There some evidence, which is consistent with our result here, is also given that there may exist periodic multivortices in the full self-dual Chern–Simons Higgs model when the basic lattice cell is sufficiently large so that its approximation by the gauged Schrödinger equation becomes effective.

In our problem, the self-duality condition requires the specific assumption that the Higgs potential be of a ϕ^6 type which limits, of course, the applicability of the theory to general situations. On the other hand, however, the mathematics here is certainly richer than the classical self-dual Abelian Higgs model where the potential

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