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The Algebra of Non-Local Charges in Non-Linear Sigma Models

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Abstract: We obtain the exact classical algebra obeyed by the conserved non-local charges in bosonic non-linear sigma models. Part of the computation is specialized for a symmetry group $O(N)$. As it turns out the algebra corresponds to a cubic deformation of the Kac–Moody algebra. We generalize the results for the presence of a Wess-Zumino term. The algebra is very similar to the previous one, now containing a calculable correction of order one unit lower. The relation with Yangians and the role of the results in the context of Lie-Poisson algebras are also discussed.

1. Introduction

In general, quantum field theoretic models where non-perturbative computations are known, contain an infinite number of conservation laws [1, 2]. In fact, the solvability of several exact S-matrices in two-dimensional models can be traced back to the Yang–Baxter relations [3, 4], which in turn are a direct consequence of the conservation of higher powers of the momentum. Alternatively, there is an infinite number of non-local conservation laws in most of these models as well [2, 5]. Both sets of conserved quantities can be related to the existence of a Lax pair in the theory: demanding compatibility of the Lax pair, one arrives at conserved charges as functions of the so-called spectral parameter implying, after Taylor expansion, an infinite number of conservation laws.

Another set of models containing an infinite number of conserved quantities are the two-dimensional conformally invariant theories [6, 7]. The Virasoro generators are a generalization of the energy-momentum-conserved charges. Defining

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