Finite-Dimensional Representations of the Quantum Superalgebra $U_q[gl(n/\text{m})]$ and Related q-Identities

T.D. Palev[*,2,3], N.I. Stoilova[2,3], J. Van der Jeugt[2**]

[1] International Centre for Theoretical Physics, I-34100 Trieste, Italy
[2] Department of Applied Mathematics and Computer Science, University of Ghent, Krijgslaan 281-S9, B-9000 Gent, Belgium
[3] Institute for Nuclear Research and Nuclear Energy, 1784 Sofia, Bulgaria (permanent address)

Abstract: Explicit expressions for the generators of the quantum superalgebra $U_q[gl(n/m)]$ acting on a class of irreducible representations are given. The class under consideration consists of all essentially typical representations: for these a Gel'fand-Zetlin basis is known. The verification of the quantum superalgebra relations to be satisfied is shown to reduce to a set of q-number identities.

1. Introduction

This paper is devoted to the study of a class of finite-dimensional irreducible representations of the quantum superalgebra $U_q[gl(n/m)]$. The main goal is to present explicit actions of the $U_q[gl(n/m)]$ generating elements acting on a Gel'fand-Zetlin-like basis, and to discuss some of the q-number identities related to these representations.

Quantum groups [5], finding their origin in the quantum inverse problem method [6] and in investigations related to the Yang-Baxter equation [15], have now become an important and widely used concept in various branches of physics and mathematics. A quantum (super)algebra $U_q[G]$ associated with a (simple) Lie (super)algebra G is a deformation of the universal enveloping algebra of G endowed with a Hopf algebra structure. The first example was given in [19,30], and soon followed the generalization to any Kac-Moody Lie algebra with symmetrizable Cartan matrix [4, 12]. For the deformation of the enveloping algebra of a Lie superalgebra we mention the case of $osp(1/2)$ [20,21], later to be extended to Lie superalgebras with a symmetrizable Cartan matrix [32] including the basic [16] Lie superalgebras [1, 2].

Representations of quantum algebras have been studied extensively, particularly for generic q-values (i.e. q not a root of unity). In this case, finite-dimensional irreducible representations of $sl(n)$ can be deformed into irreducible representations of $U_q[sl(n)]$ [13], and it was shown that one obtains all finite-dimensional irreducible

---

*E-mail: palev@bgearn bitnet
**Research Associate of the N F W O (National Fund for Scientific Research of Belgium). E-mail: Joris VanderJeugt@rug ac.be