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Dual Isomonodromic Deformations and Moment Maps to Loop Algebras

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Abstract: The Hamiltonian structure of the monodromy preserving deformation equations of Jimbo *et al* [JMMS] is explained in terms of parameter dependent pairs of moment maps from a symplectic vector space to the dual spaces of two different loop algebras. The nonautonomous Hamiltonian systems generating the deformations are obtained by pulling back spectral invariants on Poisson subspaces consisting of elements that are rational in the loop parameter and identifying the deformation parameters with those determining the moment maps. This construction is shown to lead to "dual" pairs of matrix differential operators whose monodromy is preserved under the same family of deformations. As illustrative examples, involving discrete and continuous reductions, a higher rank generalization of the Hamiltonian equations governing the correlation functions for an impenetrable Bose gas is obtained, as well as dual pairs of isomonodromy representations for the equations of the Painlevé transcendents P_V and P_{VI}.

1. Monodromy Preserving Hamiltonian Systems

The following integrable Pfaffian system was studied by Jimbo, Miwa, Môri and Sato in [JMMS]:

$$dN_i = -\sum_{\substack{j=1 \\ j\neq i}}^{n} [N_i, N_j]\, d\log(\alpha_i - \alpha_j) - [N_i, d(\alpha_i Y) + \Theta] \,. \tag{1.1}$$

Here $\{N_i(\alpha_1, \ldots, \alpha_n, y_1, \ldots, y_r)\}_{i=1,\ldots,n}$ is a set of $r \times r$ matrix functions of $n + r$ (real or complex) variables $\{\alpha_i, y_a\}_{\substack{i=1,\ldots,n \\ a=1,\ldots,r}}$, Y is the diagonal $r \times r$ matrix

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