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Classical Intertwiner Space and Quantization

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Abstract: Given two symplectic realizations, a symplectic manifold called the classical intertwiner space is introduced as a classical analogue of an intertwiner space of representations of an associative algebra. We describe explicitly how a quantum data on realizations induces a quantum data on their classical intertwiner space.

1. Introduction

Let G be a compact Lie group and X a Hamiltonian G-space. G is thus considered as a symmetry group of the classical space X. According to the "creed" of geometric quantization, if the classical space X is quantized to a quantum phase space (i.e., a Hilbert space), G becomes a symmetry group of the corresponding quantum space. This idea has in fact inspired many significant results in mathematical physics, among them orbit method of group representations and theory of geometric quantization [9, 19].

The intertwiner space between two representations $\rho_1: G \to \mathrm{End}(V_1)$ and $\rho_2: G \to \mathrm{End}(V_2)$ is, by definition, $\mathrm{Hom}_G(V_2, V_1)$, the space of all G-equivariant linear maps from V_2 to V_1. When ρ_1 is irreducible, the dimension of $\mathrm{Hom}_G(V_2, V_1)$ is usually called the multiplicity of ρ_1 in ρ_2, a fundamental concept in representation theory. The classical counterpart of $\mathrm{Hom}_G(V_2, V_1)$ is closely related to the so-called Marsden–Weinstein reduction [16]. In fact, the classical intertwiner space, for any symplectic homogeneous spaces X_1 and X_2, is defined as the symplectic (or Marsden–Weinstein) reduced space $(\overline{X_2} \times X_1)_0$. A remarkable result of Guillemin–Sternberg [5] asserts that for a Kähler manifold, the geometric quantization of classical intertwiner space is isomorphic to the intertwiner space of the corresponding representations.

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