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Long Time Behavior for the Equation of Finite-Depth Fluids

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Abstract: In this paper we study the Cauchy problem for the generalized equation of finite-depth fluids

$$\partial_t u - G(\partial_x^2 u) - \partial_x \left(\frac{u^p}{p}\right) = 0 ,$$

where $G(\cdot)$ is a singular integral, and p is an integer larger than 1. We obtain the long time behavior of the fundamental solution of linear problem, and prove that the solutions of the nonlinear problem with small initial data for $p > 5/2 + \sqrt{21}/2$ are decay in time and freely asymptotic to solutions of the linear problem. In addition we also study some properties of the singular integral $G(\cdot)$ in $L^q(R)$ with $q > 1$.

1. Introduction

In this paper we shall consider the Cauchy problem for the generalized equation of finite-depth fluids

$$\partial_t u - G(\partial_x^2 u) + \partial \left(\frac{u^p}{p}\right) = 0 , \tag{1}$$

where p is an integer larger than 1, $G(f) = \lim_{\varepsilon\to 0} \int_{|y| \geqq \varepsilon > 0} f(x-y)K(y)dy$ with $K(y) = \frac{1}{2\delta}$ (coth $\frac{\pi y}{2\delta}-$ sign y) is a singular integral, here δ is a positive real which characterizes the depth of the fluid layer. Equation (1) was first derived by Joseph [5, 10] to describe the propagation of internal waves in the stratified fluid of finite depth. It is known [1] that Eq. (1) reduces to the nonlinear Korteweg–de Vries (KdV) equation

$$\partial_t u - \partial_x^3 u + \partial \left(\frac{u^p}{p}\right) = 0 ,$$

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