Commun. Math. Phys. 162, 591–631 (1994)

Communications in
Mathematical
Physics

Selberg Supertrace Formula for Super Riemann Surfaces

III. Bordered Super Riemann Surfaces

Christian Grosche★

International School for Advanced Studies (ISAS), Via Beirut 4, I-34014 Trieste, Italy

Received: 26 October 1992/in revised form: 8 November 1993

Abstract: This paper is the third in a sequel to develop a super-analogue of the classical Selberg trace formula, the Selberg supertrace formula. It deals with bordered super Riemann surfaces. The theory of bordered super Riemann surfaces is outlined, and the corresponding Selberg supertrace formula is developed. The analytic properties of the Selberg super zeta-functions on bordered super Riemann surfaces are discussed, and super-determinants of Dirac-Laplace operators on bordered super Riemann surfaces are calculated in terms of Selberg super zeta-functions.

I. Introduction

It took a long time before physicists acknowledged the true value of the Selberg trace formula as introduced by A. Selberg in his famous paper [62]. The original attempt of Selberg to formulate his trace formula was based on number theoretical considerations, and in fact there is a close relationship between the areas of analytic number theory, eigenvalues of Laplacians on Riemann surfaces and the Selberg trace formula (see e.g. [36, 63]). In particular Selberg was interested to study the analytic properties of a function closely related to the trace formula, the Selberg zeta-function.

Physicists, however, have other objectives: they want to learn something about the spectrum of a model, or they want to calculate determinants, say. The latter approach to the use of the Selberg trace formula appears in quantum field theory on Riemann surfaces, i.e. in the Polyakov approach [20–23, 55, 56] to (bosonic-, fermionic- and super-) string theory. In the perturbation expansion of the Polyakov path integral one is left with a summation over all topologies of world sheets a string can sweep out, and an integral over the moduli space of Riemann surfaces. This picture is true for bosonic strings (BS) as well as for fermionic strings (FS). The partition function turns out to be for open as well as closed bosonic strings corresponding to a topology

★ Address from August 1993: II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chaussee 149, D-22761 Hamburg, Germany