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Positive Lyapunov Exponents for a Class of Ergodic Schrödinger Operators

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Abstract: We present a simple method to estimate the Lyapunov exponent $\gamma(E)$ for the system

$$-(\psi_{j+1}+\psi_{j-1})+v_j(\omega)\psi_j=E\psi_j\,,$$

where $\{v_j(\omega)\}_{\omega\in\Omega}$ is an ergodic family of potentials defined for $j\in\mathbb{Z}$. We assume that there is a constant $\zeta>2$ and large positive integers l, L such that for almost every ω and every E there is an infinite sequence of disjoint intervals $J_n\subset\mathbb{Z}$ with the following properties:

1) The length of each interval is larger than $2l$.
2) The distance between any two adjacent intervals is less than L.
3) $|v_j(\omega)-E|\geqq\zeta$ for $j\in\bigcup_n J_n$.

Under these conditions we prove that

$$\text{meas}\{E\colon\gamma(E)=0\}\leqq Be^{-\beta l/6}\,.$$

where β and B are positive constants and "meas" refers to Lebesgue measure.

I. Introduction

In this paper we obtain the following result:

Theorem. *Consider the finite difference Schrödinger equation*

$$-(\psi_{j+1}+\psi_{j-1})+v_j(\omega)\psi_j=E\psi_j\,. \tag{1}$$

where $\{v_j(\omega)\}_{\omega\in\Omega}$ is an ergodic family of potentials defined for $j\in\mathbb{Z}$. Let $\gamma(E)$ be the Lyapunov exponent for (1) *and let $A\equiv\{E\in\mathbb{R}:\gamma(E)=0\}$. Suppose that there is a constant $\zeta>2$ and large positive integers l, L such that for almost every ω and every E there is an infinite sequence of disjoint intervals*

$$J_n=[a_n-l_n,\ a_n+l_n]\subset\mathbb{Z}$$