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Quantized Knizhnik–Zamolodchikov Equations, Quantum Yang–Baxter Equation, and Difference Equations for q-Hypergeometric Functions

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Abstract: The $s\ell_2$ quantized Knizhnik–Zamolodchikov equations are solved in q-hypergeometric functions. New difference equations are derived for general q-hypergeometric functions. The equations are given in terms of quantum Yang–Baxter matrices and have the form similar to quantum Knizhnik–Zamolodchikov equations for quantum affine algebras introduced by Frenkel and Reshetikhin.

Introduction

The Knizhnik–Zamolodchikov (KZ) differential equation is the fundamental differential equation of the Conformal Field Theory with very rich mathematical structures. The KZ equation connects representation theories of Lie algebras and quantum groups [KZ, D, K, KL, SV, V]. Quantization of the KZ equation is of great importance. It is expected that the quantized KZ equation also will connect two representation theories. The first is presumably the theory of representations of quantum groups and the second is the theory of representations of a yet undefined structure that may be called "a double quantum group" or "an elliptic quantum group," see [FR].

The KZ equation coincides with the Gauss–Manin differential equation for general hypergeometric functions [SV]. General hypergeometric functions are integrals of special hypergeometric forms over suitable cycles depending on parameters. The special hypergeometric forms are naturally identified with objects of the representation theory of Lie algebras, the cycles are naturally identified with objects of the representation theory of quantum groups, the integration of hypergeometric forms over cycles gives a natural correspondence between representation theories of Lie algebras and quantum groups [FW, V].

There are two ways to quantize the KZ equation: through representation theory and through geometry. The quantization through representation theory

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