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Erdős-Rényi Laws for Gibbs Measures

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Abstract: Can one detect a phase transition from a single, large sample of a Gibbs measure? What information does one get on the other Gibbs distributions with the same potential? We approach these questions via Erdős-Rényi laws. In particular we prove almost-sure limit theorems for sets of empirical distributions of sub-samples of the given one: for suitable sub-samples size this set converges to the set of stationary Gibbs measures. Moreover we formulate Erdős-Rényi laws for general families of random variables with suitable large deviation principles.

I. Introduction

On a single realization of a random field on the lattice observed in a large box, one can see smaller windows where the sample shows a large deviation from its typical behavior. The smallest the size of the windows, the most unlikely the deviation. This is the underlying idea of Erdős-Rényi laws, which are well known from statisticians for independent identically distributed random variables.

In this paper, we prove Erdős-Renyi type laws for Gibbs distributions, with a particular emphasis on (first-order) phase transition. Let us illustrate our results in the case of a real valued, finite range interaction Gibbs random field P. Consider the average spin $M_\Lambda(\omega) = \frac{1}{|\Lambda|} \sum_{i \in \Lambda} \omega_i$ of the sample $\omega = (\omega_i)_i$ on a cubic box Λ with cardinality $|\Lambda|$. It is well known that for large Λ, $P\{M_\Lambda \geq x\}$ behaves approximately like $\exp\{-|\Lambda|\lambda(x)\}$, with $\lambda(x) > 0$ if x is larger than some number. Let us observe the sample on a box Λ; for cubic windows $\Lambda' \subset \Lambda$ the Erdős-Rényi statistics $M^+_{\Lambda,\Lambda'}$ is the largest average spin $M_{i+\Lambda'}$ among all the translates $i + \Lambda'$ of Λ' which are included in Λ. Then, for all such x the Erdős-Rényi law proved in this paper states that

$$M^+_{\Lambda,\Lambda'} \longrightarrow x \quad P\text{-a.s.} \tag{1.1}$$

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