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A Generalized Simple Random Walk in One Dimension Related to the Gaussian Polynomials

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Abstract: A generalization of the relation between the simple random walk on a regular lattice and the diffusion equation in a continuous space is described. In one dimension we consider a random walk of a walker with exponentially decreasing mobility with respect to time. It has an exact solution of the conditional probability, that is expressed in terms of the Gaussian polynomials, a generalization of binomial coefficients. Taking a suitable continuum limit we obtain the corresponding transport equation from the recursion relation of the discrete random walk process. The kernel of this differential equation is also directly obtained from that conditional probability by the same continuum limit.

1. Introduction

It is well known that the diffusion equation in a continuous space is obtained as a continuum limit of the simple random walk process on a regular lattice in arbitrary dimension (see, for example Chapter 1 of [4]). Let us consider the one-dimensional case. The diffusion (or heat) equation

$$\frac{\partial P}{\partial t}(x, t)=\frac{1}{4}\frac{\partial^2 P}{\partial x^2}(x, t)\,, \tag{1.1}$$

has the kernel

$$P(x, t)=\frac{1}{\sqrt{\pi t}}\,e^{-x^2/t}\,. \tag{1.2}$$

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