Commun. Math Phys. 162, 147–173 (1994)

Communications in
Mathematical
Physics

Symplectic Structures Associated to Lie-Poisson Groups

A. Yu. Alekseev [1,2,*] **A. Z. Malkin** [2,**,***]

[1] Laboratoire de Physique Théorique et Hautes Énergies****, Paris, France*****
[2] Institute of Theoretical Physics, Uppsala University, Box 803 S-75108, Uppsala, Sweden

Received: 1 April 1993/in revised form: 14 October 1993

Abstract: The Lie-Poisson analogues of the cotangent bundle and coadjoint orbits of a Lie group are considered. For the natural Poisson brackets the symplectic leaves in these manifolds are classified and the corresponding symplectic forms are described. Thus the construction of the Kirillov symplectic form is generalized for Lie-Poisson groups.

Introduction

The method of geometric quantization [9] provides a set of Poisson manifolds associated to each Lie group G. The dual space $\mathscr{G}^*$ of the corresponding Lie algebra $\mathscr{G}$ plays an important role in this theory. The space $\mathscr{G}^*$ carries the Kirillov-Kostant Poisson bracket which mimics the Lie commutator in $\mathscr{G}$. Having chosen a basis $\{\varepsilon^a\}$ in $\mathscr{G}$, we can define structure constants f_c^{ab}:

$$[\varepsilon^a, \varepsilon^b] = \sum_c f_c^{ab} \varepsilon^c , \tag{1}$$

where $[\,,]$ is the Lie commutator in $\mathscr{G}$. On the other hand, we can treat any element ε^a of the basis as a linear function on $\mathscr{G}^*$. The Kirillov-Kostant Poisson bracket is defined so that it resembles formula (1):

$$\{\varepsilon^a, \varepsilon^b\} = \sum_c f_c^{ab} \varepsilon^c . \tag{2}$$

The Kirillov-Kostant bracket has two important properties:

---

* On leave of absence from LOMI, Fontanka 27, St Petersburg, Russia
** Supported in part by a Soros Foundation Grant awarded by the American Physical Society
*** On leave of absence from St Petersburg University
**** Unité Associée au C N.R S., URA 280
***** LPTHE, Paris-VI, Tour 16 – 1er étage, 4 place Jussieu, F-75252 Paris Cedex 05, France