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An Investigation of the Limiting Behavior of Particle-Like Solutions to the Einstein-Yang/Mills Equations and a New Black Hole Solution

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Abstract: A mathematical investigation of the limiting behavior of particle-like solutions of Einstein-Yang-Mills equations leads to a discovery of a new type of black hole solution.

1. Introduction

In the paper [4], we proved the existence of a countably infinite number of smooth, static, spherically symmetric solutions of the Einstein-Yang/Mills equations (EYM) with $SU(2)$ gauge group (first observed by Bartnik and McKinnon in [1]). These solutions are indexed by a bounded real parameter λ_n. Our first objective here is to study the limiting solution corresponding to the parameter value $\bar{\lambda}$, where

$$\bar{\lambda} = \lim \lambda_n \,,$$

and to describe some of the rather interesting mathematical properties of this solution. In particular, we prove that this solution is the first "crashing" solution, (in the sense that a metric coefficient becomes singular) and that this solution crashes at $r = 1$. Next we show that this degenerate orbit admits (at least) one pseudo-continuation (PC) defined for all $r > 1$. The concatenation of the $\bar{\lambda}$-orbit, defined for $r < 1$, and the "PC orbit" defined for $r > 1$, $(w(r), w'(r), A(r))$, satisfies, (for some subsequence $\{\lambda_{n_j}\}$ of $\{\lambda_n\}$),

$$\lim_{j\to\infty} (w_{n_j}(r, \lambda_{n_j}), w'_{n_j}(r, \lambda_{n_j}), A_{n_j}(r, \lambda_{n_j})) = (w((r), w'(r), A(r)) \,.$$

In addition, $\lim_{r\nearrow 1}(A(r, \bar{\lambda}), A'(r, \bar{\lambda}), w(r, \bar{\lambda})) = (0, 0, 0) = \lim_{r\searrow 1}(A(r), A'(r), w(r))$, but neither $\lim_{r\searrow 1} w'(r)$, nor $\lim_{r\nearrow 1} w'(r, \bar{\lambda})$ exists. Now although the λ_n-orbits are all particle-like solutions of the EYM equations, the PC orbit in $r > 1$ can be interpreted as a

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