Commun. Math. Phys. 160, 391–429 (1994)

Communications in
Mathematical
Physics

GLM Equations, Tau Function and Scattering Data

Max R. Niedermaier

Max-Planck-Institut für Physik, Werner Heisenberg Institut, Föhringer Ring 6, D-80805 Munich, Germany

Received: 22 March 1993

Abstract: The direct and the inverse scattering problem for affine Toda/mKdV systems is addressed and is found to develop non-standard features within the framework of the inverse scattering method. A solution scheme based on the tau function formalism is described. The inverse problem is shown to be equivalent to a set of decoupled, scalar Gelfand-Levitan-Marchenko-type equations. The Fredholm-Grothendieck determinants of the latter are shown to define tau-functions in the sense of the Kyoto School. In particular, a simple monodromy formula allows the derivation of trace identities.

1. Introduction

For many integrable field theories the inverse scattering method (ISM) provides the most complete and physically compelling insight into the structure of the classical phase space. The basic discovery, dating back to Gardner, Greene, Kruskal and Miura [9], is that a generic solution can be parametrized through the scattering data of some auxiliary linear system (generalised Schrödinger equation). Although initially designed for the specific example of the KdV equation, the principle turned out to be systematically applicable to a wide range of systems (see e.g. the book [21] for an exposition). Moreover the scattering data were found to be related to action-angle variables, turning these models into infinite dimensional completely integrable Hamiltonian systems [8].

An independent development was initiated by the observation of Hirota that many of these nonlinear equations could be bilinearized by a suitable change of variables, so that a direct construction of solutions became possible [12]. In addition, these variables ("τ-functions") were discovered by the Kyoto School to describe the orbits of affine Lie groups in a particular realization [4, 14]. This lead to a systematic construction and classification scheme for integrable systems in terms of the data associated with some affine Lie algebra. An overlapping, presumably yet broader algebraic scheme was developed by Drinfeld and Sokolov [6] (see also [5, 13]). We