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Superderivations of C^*-Algebras Implemented by Symmetric Operators

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Abstract: The paper studies unbounded symmetric and dissipative implementations (S, G) of *-superderivations δ of C^*-algebras $\mathfrak{U}$. It associates with them representations π_S^δ of the domains $D(\delta)$ of δ on the deficiency spaces $N(S)$ of the symmetric operators S. A link is obtained between the deficiency indices $n_\pm(S)$ of S and the dimensions of irreducible representations of $\mathfrak{U}$. For the case when (S, G) is a maximal implementation and $\max(n_\pm(S)) < \infty$, some conditions are given for the representation π_s^δ to be semisimple and to extend to a bounded representation of $\mathfrak{U}$.

1. Introduction

Let $\mathfrak{U}$ be a C^*-algebra and ϱ be a *-representation of $\mathfrak{U}$ on a Hilbert space $\mathfrak{H}$. Let δ be a linear closed mapping from a dense *-subalgebra $D(\delta)$ of $\mathfrak{U}$ into the algebra $B(\mathfrak{H})$ of all bounded operators on $\mathfrak{H}$ such that, for $A \in D(\delta)$,
(i) $\delta(AB) = \delta(A)\varrho(B) + \varrho(\varphi(A))\delta(B)$,
(ii) $\delta(\varphi(A)^*) = \delta(A)^*$,

where φ is an automorphism of $D(\delta)$. Then δ a closed *-*superderivation of* $\mathfrak{U}$ *relative to the pair* (ϱ, φ). A pair (S, G), where S is a densely defined closed operator on $\mathfrak{H}$, S^* is its adjoint and G is a bounded operator on $\mathfrak{H}$ such that $G^{-1} \in B(\mathfrak{H})$, implements δ if, for $A \in D(\delta)$,

$$\varrho(\varphi(A)) = G^{-1}\varrho(A)G\,, \tag{1}$$

$$GD(S) = D(S) \quad \text{and} \quad GD(S^*) = D(S^*)\,, \tag{2}$$

$$\varrho(A)D(S) \subseteq D(S) \quad \text{and} \quad \delta(A)|_{D(S)} = i(S\varrho(A) - G^{-1}\varrho(A)GS)|_{D(S)}\,. \tag{3}$$

If a pair (T, G) also implements δ and T extends S, then (T, G) is a δ-extension of (S, G). If S is symmetric and G is selfadjoint, (S, G) is a symmetric implementation of δ. If (S, G) has no symmetric δ-extensions, it is a maximal symmetric implementation of δ.