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The Relation Between Quantum W Algebras and Lie Algebras

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Abstract: By quantizing the generalized Drinfeld-Sokolov reduction scheme for arbitrary sl_2 embeddings we show that a large set $\mathscr{W}$ of quantum W algebras can be viewed as (BRST) cohomologies of affine Lie algebras. The set $\mathscr{W}$ contains many known W algebras such as W_N and $W_3^{(2)}$. Our formalism yields a completely algorithmic method for calculating the W algebra generators and their operator product expansions, replacing the cumbersome construction of W algebras as commutants of screening operators. By generalizing and quantizing the Miura transformation we show that any W algebra in $\mathscr{W}$ can be embedded into the universal enveloping algebra of a semisimple affine Lie algebra which is, up to shifts in level, isomorphic to a subalgebra of the original affine algebra. Therefore *any* realization of this semisimple affine Lie algebra leads to a realization of the W algebra. In particular, one obtains in this way a general and explicit method for constructing the free field realizations and Fock resolutions for all algebras in $\mathscr{W}$. Some examples are explicitly worked out.

1. Introduction

W algebras were introduced by Zamolodchikov as a new ingredient in the classification program of conformal field (CFT) theories [1] (for a recent review see [2]). As is well known such a classification would correspond to a classification of all possible perturbative groundstates of string theory. However CFT and W algebras have been shown to be related to several other areas of research as well such as integrable systems, 2D critical phenomena and the quantum Hall effect. W symmetries are therefore an interesting new development in theoretical physics and it is the purpose of this paper to provide a step towards understanding their meaning and structure.

The point of view that we shall develop in this paper is that the theory of W algebras is closely related to the theory of Lie algebras and Lie groups. The

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