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On the Critical Behaviour of Dyson's Quantum Hierarchical Models

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Abstract: We study the probability distribution of the appropriately scaled square of the total spin for critical asymptotically hierarchical quantum models and show that it converges, as the number of spins tends to infinity, to the same function related to the corresponding classical systems. Thus, we exhibit explicitly a property of statistical mechanical systems which, at the critical point, does not depend whether one uses a classical or quantum mechanical description.

1. Introduction

It is widely believed that the critical behaviour of statistical mechanical system is independent of the way (classical or quantum mechanical) used to describe it. Sewell [8] has shown quite generally that the long distance behaviour of critical quantum systems is classical, but the question on the relation between this classical behaviour and the one corresponding to the classical version of the system analyzed was not considered.

In the last decade, rigorous renormalization group methods have been developed, which permitted, among other things, a deep understanding of some classical critical systems, particularly those having infrared asymptotically free behaviour [6]. On the other hand, no such progress has been achieved to treat quantum critical systems in general – a task much more difficult than the classical case because of the appearance of non-commutative objects.

In this paper we consider a special class of spin systems which can be analyzed both classically and quantum mechanically: the asymptotically hierarchical models in the terminology of [9]. These are generalizations of Dyson's hierarchical models, originally introduced in [4]. The classical version of these models, with the spins taking values ± 1, at the critical point was first studied in a rigorous way in the fundamental paper by Bleher and Sinai [3]. Vector valued classical spins at low temperatures were analyzed by Bleher and Major in [1, 2] and by Schor and O'Carroll in [7], using a different hierarchical model. Here we consider instead quantum mechanical spin $\frac{1}{2}$ systems and compare them with the corresponding classical three dimensional vector models. Specifically, we study the probability