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Shift Operators for the Quantum Calogero–Sutherland Problems via Knizhnik–Zamolodchikov Equation

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Abstract: We give a natural interpretation of the shift operators for Calogero–Sutherland quantum problem via KZ equation using Matsuo–Cherednik mappings. The explicit formulas for the inversions of these mappings and versions of shift operators for KZ equations are also found. As an application we show that the shift operator can be described via a factorization problem for an appropriate quantum integral (discriminant) of the Calogero system.

Introduction

The *Calogero system* [1] describes the motion of N particles on the line interacting with the potential $U_g^c(x) = gu^c(x)$,

$$u^c(x) = \sum_{i \neq j}^{N} \frac{1}{(x_i - x_j)^2}, \tag{0.1}$$

x_i are the coordinates of the points, g is a coupling constant. The value of g is not essential in the classical case but it is in the quantum case. For $N = 2$ the corresponding Schrödinger equation

$$(-\Delta + U_g^c(x))\psi = E\psi, \tag{0.2}$$

$\Delta = \partial_1^2 + \cdots + \partial_N^2$, $\partial_i \equiv \partial/\partial x_i$, can be reduced to the one-dimensional one:

$$\left(-\frac{d^2}{dy^2} + \frac{g}{y^2}\right)\varphi = \lambda\varphi, \quad y = x_1 - x_2 .$$

If we introduce a new constant k, such that

$$k(k-1) = g, \tag{0.3}$$