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Existence of Self-Similar Blow-Up Solutions for Zakharov Equation in Dimension Two. Part I

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Abstract: We consider the Zakharov equation in space dimension two

$$\begin{cases} iu_t = -\Delta u + nu\,, \\ \dfrac{1}{c_0^2}\, n_{tt} = \Delta n + \Delta |u|^2\,. \end{cases}$$

We prove the existence of blow-up solutions (stable "self-similar" blow-up solutions) for this problem and we study various properties of these solutions.

I. Introduction

In this paper, we consider the Zakharov system in space dimension two:

$$(\mathrm{I}_{c_0}) \qquad \begin{cases} iu_t = -\Delta u + nu\,, & (1.1) \\ \dfrac{1}{c_0^2}\, n_{tt} = \Delta n + \Delta |u|^2\,, & (1.2) \\ u(0) = \phi_0\,, \quad n(0) = n_0\,, \quad n_t(0) = n_1\,, \end{cases}$$

where $c_0 > 0$, Δ is the Laplace operator on $\mathbb{R}^2$, $u : [0,T)\times\mathbb{R}^2 \to \mathbb{C}$, $n : [0,T)\times\mathbb{R}^2 \to \mathbb{R}$ and ϕ_0, n_0, n_1 are initial data.

This model is often used to describe Langmuir waves in plasmas when the electric field is one dimensional. u represents the envelope of the electric field and n is the large scale flucatuation of the ionic density. We remark that the subsonic limit of these equations ($c_0 \to +\infty$) is formally

$$(\mathrm{I}_\infty) \qquad \begin{aligned} iu_t &= -\Delta u - |u|^2 u\,, & (1.3) \\ u(0) &= \phi_0\,. & (1.4) \end{aligned}$$

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