

Beta Function and Schwinger Functions for a Many Fermions System in One Dimension. Anomaly of the Fermi Surface

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Abstract: We present a rigorous discussion of the analyticity properties of the beta function and of the effective potential for the theory of the ground state of a one dimensional system of many spinless fermions. We show that their analyticity domain as a function of the running couplings is a polydisk with positive radius bounded below, uniformly in all the cut offs (infrared and ultraviolet) necessary to give a meaning to the formal Schwinger functions. We also prove the vanishing of the scale independent part of the beta function showing that this implies the analyticity of the effective potential and of the Schwinger functions in terms of the bare coupling. Finally we show that the pair Schwinger function has an anomalous long distance behaviour.

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