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Multivoice Littlewood–Paley–Meyer Wavelets and Diagonal Dominated Pseudodifferential Operators

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Abstract: Given a pseudodifferential operator $\varepsilon(p)$ satisfying certain growth and smoothness conditions in momentum space, we construct a wavelet basis of $L^2(\mathbb{R}^d)$ in which the corresponding matrix is diagonal dominated with arbitrarily small prefactor.

1. Introduction

Problems arising in several branches of mathematical physics including quantum field theory, fluid dynamics and semiclassical analysis require some sort of multi-scale analysis. The techniques based on the tree expansion in quantum field theory and on pseudodifferential calculus have recently been complemented by a new tool: the wavelet bases of $L^2(\mathbb{R}^d)$ discovered by Meyer, Lemarié, Daubechies, Mallat and others. We refer to Meyer's books [M] for a review of these results and for further references.

The prototypical basis found by Meyer in 1988 is given in dimension one by a family of functions $\psi_x(\xi) \in L^2(\mathbb{R})$, $x = (s(x), \xi(x))$, $s(x) \in \mathbb{Z}$ and $\xi(x) \in 2^{-s(x)}\mathbb{Z}$, which is generated by one "mother" function $\psi(\xi) \in L^2(\mathbb{R})$ so that

$$\psi_x(\xi) = 2^{\frac{s(x)}{2}} \psi(2^{s(x)}(\xi - \xi(x))) \tag{1.1}$$

and

$$\operatorname{supp} \hat{\psi} = \left\{ p \in \mathbb{R}: \frac{\pi}{3} \leqq |p| \leqq \frac{4\pi}{3} \right\}. \tag{1.2}$$

Meyer's wavelets also have a "father" $\phi(\xi)$ in terms of which $\psi(\xi)$ is defined and which helps to generate higher dimensional wavelets by the method of tensor products. In Sect. 2, we give a selfcontained review of these constructions. This particular basis has been named by Meyer after Littlewood and Paley. We find it thus natural to call these basis functions "LPM-wavelets."