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Charge Deficiency, Charge Transport and Comparison of Dimensions

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Dedicated to Elliott Lieb on the occasion of his 60th birthday

Abstract: We study the relative index of two orthogonal infinite dimensional projections which, in the finite dimensional case, is the difference in their dimensions. We relate the relative index to the Fredholm index of appropriate operators, discuss its basic properties, and obtain various formulas for it. We apply the relative index to counting the change in the number of electrons below the Fermi energy of certain quantum systems and interpret it as the charge deficiency. We study the relation of the charge deficiency with the notion of adiabatic charge transport that arises from the consideration of the adiabatic curvature. It is shown that, under a certain covariance, (homogeneity), condition the two are related. The relative index is related to Bellissard's theory of the Integer Hall effect. For Landau Hamiltonians the relative index is computed explicitly for all Landau levels.

1. Introduction

An interesting observation that emerged in the last decade is that charge transport in quantum mechanics, in the absence of dissipation, often lends itself to geometric interpretation. A good part, but not all, of this research has been motivated by, and applied to, the integer and fractional Hall effect [2, 8, 11, 17, 20, 26, 32, 34, 35, 38, 44].

The framework that will concern us here is that of (non-relativistic) quantum mechanics. Within this framework common models of the integer Hall effect are Schrödinger operators associated with non-interacting electrons in the plane, with (constant) magnetic field perpendicular to the plane and random (or periodic) potential. The Hall conductance has been related to a Fredholm Index by Bellissard [5], and to a Chern number by Thouless, Kohmoto, Nightingale and den-Nijs [40]. The Fractional

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